Transient Behavior of Single-phase Natural-circulation Loop Systems

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A useful method is reported for calculating temperatures and rates of flow in the unsteady-state operation of natural-circulation loops in single phase. A one-dimensional mathematical model is used with the assumptions that (1) at any instant the volumetric rate of flow is constant around the loop and (2) steady-state friction factors can be applied in transient operations. The loop, consisting of a heat source, heat sink, hot leg, cold leg, and connecting piping, is divided into a number of finite increments. The transient behavior is calculated by the iterative application of the finite-difference momentum and energy balances. Numerical computations made for several cases of transient operations were carried out with the aid of the Standard Eastern Automatic Computer (SEAC).

Comparisons of predicted with actual performances were checked by use of two experimental loops employing water and found satisfactory.

A number of practical problems arising in boiler and nuclear-reactor-coolant designs have demonstrated the desirability of establishing a method for the prediction of temperatures and rates of flow in the transient operation of natural-circulation loops. To the authors' knowledge, this paper presents the first published comparisons of calculated and experimental transient behavior of a natural-circulation loop involving liquid water as the circulating fluid. The principal objective of this study was to determine the reliability of the calculations based upon numerical solutions of finite-difference energy and flow equations. Solutions were obtained through the use of the Standard Eastern Automatic Computer (SEAC).

Figures 1 and 2 illustrate the natural-circulation loops, and Table 1 lists the conditions for the transient operation of the loops for several problems computed. Details of the loops and the conditions of operation are given in later sections, along with an illustration of the complete system of equations used for one problem. The complexities of the physical problems were appreciated at the outset, and thus it was desirable to start with a simplified mathematical model and to introduce only needed refinements. For the numerical transient evaluations, it was necessary to calculate and experimentally to verify the loop parameters for steady-state flow. The refinement of the loop parameters on the transient calculations is presented, for *example*, as revisions of computer problem 3. (*See* Table 2.)

PREVIOUS INVESTIGATIONS

Of the extensive literature on transient flow, it is of interest to note that Jenny(*9*), among others, pre-

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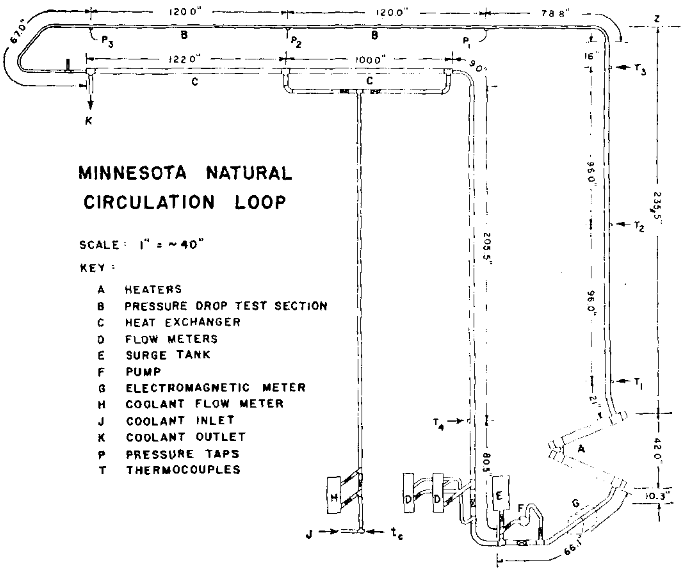

Fig. 1. Minnesota natural-circulation loop.

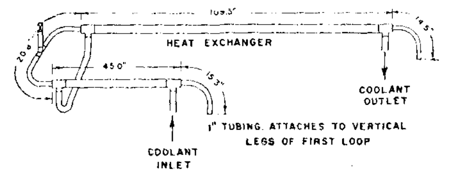

Fig. 2. Top section of second natural-circulation loop.

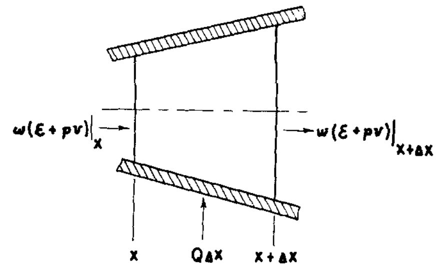

Fig. 3. Volume element of fluid for energy.

sented the derivation of the equation of motion for the unsteady turbulent flow of fluid in a horizontal tube. Accelerated flow may occur in a steady, nonuniform flow, such as at the entrance to a tube, from a reservoir wherein the boundary layer grows along the tube. Among those who have studied this problem is Langhaar(*11*). Grace(*6*) considered the oscillatory motion of a fluid in a pipe, and Valensi(*21*) was concerned with the oscillatory viscous flow in a U tube. Schonfeld(*20*), considering isothermal transient viscous flow in pipes for the case where a finite pressure differential is applied to a fluid initially at rest, found that the flow of fluid approached steady-state conditions asymptotically. He also studied forced oscillation in round tubes and natural oscillation in U tubes. The mixing-length theory of Prandtl and Von Karmen was used to study fully developed turbulent flow in round tubes and wide-open canals. Daily and Deemer(*4*) reported that steady-state friction factors were applicable to the transient flow of fluid and found that the total pressure drop across a given section could be calculated from the sum of the instantaneous frictional pressure drop and acceleration head. DeJuhasz(*5*) presented a graphical method for determining the response of a system to unbalances. Velocity and pressure may be determined as functions of time and position for isothermal systems subjected to known disturbances. Binnie

TABLE 1.—FLOW CONDITIONS FOR COMPUTER PROBLEMS

Computer problem	Initial volumetric rate of flow	Initial temp., °F.	Nature of step change	Coolant temp., °F.	Comments
1*	0	Uniform around loop 80.5	Abrupt heat input 4 heaters, 25.08 kw.	40	First loop
2	0	Uniform around loop 122.0	Abrupt heat input 4 heaters, 24.75 kw.	37.5	First loop
3	0.00137 cu. ft./sec.	144.0 in riser 77.5 in downcomer	Bottom heater, 6.30 kw. Abrupt heat input 3 heaters, 18.58 kw.	36.5	First loop
4	0	Uniform around loop 109.0	Abrupt heat input Top three heaters, 19.17 kw.	40	Second loop

*Not reported in this paper.
Revisions to computor problems pertain to changes in loop parameters and number of subdivisions. (*See* Table 2.)

TABLE 2—NATURE OF REVISIONS FOR COMPUTER PROBLEM 3

	Comments	Heat transfer coefficient: Cooler (Over-all)	Heat transfer coefficient: Vertical riser (Film)	Loop resistance	Heater lag	Heat transfer to heater walls	Heater volume, cu ft.
Initial problem		$U=35.1V$	$h=43V$	$R_l=10^6V^{1.65}$	Not taken into account	Not taken into account	0.176 (calc'd.)
First revision	Error in heat capacity of tubing walls corrected. Simple heater lag taken into account.	Same	Same	Same	$Q'=q'(1-0.189e^{-18.3V\tau})$	Same	Same
Second revision	Film heat transfer coefficient revised. Cooler coefficient revised to take into account cooler subdivisions. Loop resistance evaluated on basis of 10 gal./min. for coolant flow. Improved heater lag equation used. Heater volume measured.	$U=41.5V$	$h=58.7V$	$R_l=690{,}000V^{1.58}$	$Q'=q'(1-0.173e^{-18.3V\tau}-0.827e^{-197V\tau})$	Same	0.184 (measured)
Third revision	Heat transfer to heater walls taken into account.	Same	Same	Same	Same	Taken into account	Same
Fourth revision	Size of subdivisions nearly doubled.	Equation corrected to take into account subdivisions.	Same	Same	Same	Same	Same

NOTE.—*Same* designates condition of previous revision not changed.

(*2*) treated a somewhat similar problem but obtained an analytical solution. He was able to calculate the pressure surges which occur in long pipe lines because of the closure of a valve. Allowance was made for the effect of friction, which was assumed proportional to velocity. For cases where the friction was not directly proportional to velocity, the method gives an approximate value. Hall(*7*) considered pulsating flows and their effect on orifice and flow coefficients. The calculated accelerations of fluids in an isothermal U loop were experimentally verified by Leppert(*14*).

EXPERIMENTAL LOOPS

Two experimental loops were designed, erected, and instrumented. A schematic drawing of the first convection loop is given in Figure 1. To obtain faster accelerations, the top horizontal section of the first loop was replaced by a much shorter, larger-diameter section as illustrated in Figure 2, and this arrangement with its new method of measuring flow constituted the second circulation loop. The flowmeter for the first experimental loop was a differential pressure transmitter and recorder which measured the pressure drop across the horizontal pressure-drop test section. Though this method of evaluating the transient flow from the pressure-drop readings was cumbersome, it was found to be satisfactory for the relatively slow transient in the first circulation loop. An electromagnetic flowmeter was used to measure the instantaneous volumetric rate of flow in the second loop.

The heater consisted of four 7.5-kw. Chromalox immersion heaters arranged in a V shape as noted in Figure 1. Bare platinum–platinum-rhodium thermocouples (24 gauge) were directly exposed to the fluid to obtain maximum response. The fluid temperatures T_1 to T_4 were determined at locations noted in Figure 1 and were recorded on a Leeds and Northrup Speedomax recorder. Concentric tube heat exchangers were used to cool the flow. The general arrangements for the two loops are shown in Figures 1 and 2.

The vertical riser and downcomer sections were made of 1-in. O.D. hard-brass tubing (0.872 in. avg. I.D.), giving the loop an over-all height of about 24 ft. As shown in Figure 1, the system of valving in the bottom horizontal return section permitted runs also to be made with forced circulation by use of the pump and rotameters. Compressed air was introduced to a small surge tank (4-in.-diam. brass pipe, 11½ in. long) to regulate the pressures over a range of about 40 to 80 lb./sq.in. A minimum liquid level was maintained in the surge tank to reduce pressure fluctuations during the transient operations.

Steady-state–flow runs were made to determine the physical parameters

TABLE 3.—LOOP PARAMETERS FOR SECOND EXPERIMENTAL LOOP

Heat Transfer Coefficients	
Over-all coefficients for heat exchanger (cooler)	$U-16.1V$
3/4-in. tubing	$h_{3/4}=56.7V$
1-in. tubing	$h_1=31.7V$
Heater section	$h_h=16.0V$
Loop Resistances	
Turbulent flow	$Rl=332{,}000V^{1.64}$
Viscous flow	$Rl=2{,}500\mu V$
Dimensions and Other Values	
$D_o=1.000$ in., $D_n=0.872$ in., $\delta=0.00573$ ft., $\rho_w c_w=53.0$ B.t.u./(cu. ft.) (°F.) for 1-in. tubing	
$a_l=\dfrac{\rho}{g}\int\dfrac{dx}{A}=(\rho/g)\sum_{n-1}^{N}\dfrac{x_n}{A_n}=16{,}990\ \rho/g=32{,}200$ lb. sec. ^{2}/ft.^{5}	
1-in. tubing	$\dfrac{L}{A}=14{,}530$ ft.$^{-1}$
3/4-in. tubing	$\dfrac{L}{A}=1{,}880$
1-in. pipe	$\dfrac{L}{A}=280$
Heater	$\dfrac{L}{A}=290$
$\rho c=60.9$ B.t.u./(cu. ft.) (°F.)	Average density used

TABLE 4.—FINITE DIFFERENCE EQUATIONS FOR COMPUTER PROBLEM 4

Subdivision	
Hot leg 1-15	$T'_n=(1-197.1\ V\Delta\tau)\ T_n+168.5\ V\Delta\tau\ T_{n-1}+28.6\ V\Delta\tau\ t_n$ $t'_n=(1-105\ V\Delta\tau)\ t_n+105\ V\Delta\tau\ T_n$
Cooler 16-24	$T'_n=(1-183.1\ V\Delta\tau)\ T_n+168.5\ V\Delta\tau\ T_{n-1}+14.6\ V\Delta\tau\ t_c$
Cold leg 25-44	$T'_n=(1-192.8\ V\Delta\tau)\ T_n+164.2\ V\Delta\tau\ T_{n-1}+28.6\ V\Delta\tau\ t_n$ $t'_n=(1-105V\Delta\tau)\ t_n+105\ V\Delta\tau\ T_n$
Heater 45-48	$T'_n=(1-27.7\ V\Delta\tau)\ T_n+21.7\ V\Delta\tau\ T_{n-1}+6.0\ V\Delta\tau\ t_n$ $+0.338\ (Q_{KW})_n\ \Delta\tau\ (1-0.173\ e^{-18.3V\tau}-0.827\ e^{-0.1\tau})$ $t'_n=(1-16.5\ V\Delta\tau)\ t_n+16.5\ V\Delta\tau\ T_n$
Flow equations	$V'=V+\Delta\tau(0.0000311\ H'_l-0.0481V)$ for $V<0.00085$ $V'=V+\Delta\tau(0.0000311\ H'_l-10.32V^{1.64})$ for $V>0.00085$ $H_l=\sum_{n=1}^{48}\rho_n Z_n$
Conditions of restraint	$Z_1=-1.3475$; $Z_{2-13}=-1.4308$; $Z_{14}=-1.1075$; $Z_{15-24}=0$; $Z_{25}=0.3640$; $Z_{26-41}=1.4687$; $Z_{42}=0.1156$; $Z_{43}=0$; $Z_{44}=-0.8542$; $Z_{45-48}=-0.8750$ $\rho_n=62.404+0.4556\left(\dfrac{T_n}{100}\right)-0.9605\left(\dfrac{T_n}{100}\right)^2+0.08164\left(\dfrac{T_n}{100}\right)^3$ $n_{49}=n_1$
Operating conditions	$V=0$, $\tau=0$; $t_c=40.0$; $(Q_{KW})_{45}=0$; $(Q_{KW})_{46-49}=6.39$; $(T_{1-48})_{\tau=0}=109.0$; $(T_n)_{\tau=0}=(t_c)_{\tau=0}$

of the experimental loops. These parameters include the friction factors, over-all heat transfer coefficients for the coolers, fluid film heat transfer coefficients, total loop resistance, and heater lag. It was possible to demonstrate that simplified relations could be used for some of the parameters; for example, the correlations used for the over-all heat transfer coefficient in the cooler, the film heat transfer coefficient in the vertical riser, and the loop resistance are given in Table 2. These correlations are discussed in reference *1*, including the manner in which the number of subdivisions in the cooler affects the over-all heat transfer coefficient.

All the transient runs reported in this paper involve a sudden increase in the heat input to the loop. It was recognized that a step change in heater switch settings would not introduce instantaneously the corresponding power change to the fluid. Consequently, it was deemed desirable to introduce into the finite-difference-method calculations heater-lag equations of various degrees of complexity. The initial calculations made for all runs on the first circulation loop involved no corrections for the heater lag, and, as illustrated in Table 2, heater-lag provisions, along with other refinements in loop parameters, were then introduced into the succeeding revisions. The determination of the heater-lag time was approached from both a theoretical and an experimental point of view, and the details are given in reference *1*. It is to be noted that all refinements introduced for the heater lag were made independently of the transient operation of the natural-circulation loops and were based only on the performance of steady-state–flow runs.

TRANSIENT ANALYSIS

The transient analysis is restricted to the one-dimensional, nonsteady, rectilinear flow of real fluids. The assumption of one-dimensional flow is strictly applicable to cases where the velocity of the fluid is uniform over the entire cross section of flow. The analysis is therefore valid only for an ideal case, but in practice an average velocity may be used for a variety of velocity profiles and the problem considered ideal. It is assumed that the fluid is continuous, no sources or sinks therefore existing, and that no heat is generated or lost within the body, for instance by chemical reaction. The effects due to the conduction of heat along the longitudinal axis are assumed to be negligible.

The continuity equation may be written as

$$\frac{\partial\rho}{\partial\tau}+\frac{1}{A}\frac{\partial(\rho uA)}{\partial x}=0 \qquad (1)$$

In a system consisting of the volume element of fluid shown in Figure 3, where it is assumed that there is no shaft work, the heat input, Q, is defined as net heat added to the system per foot per second, and the symbol ϵ represents the total energy per pound of fluid, $E+(u^2/2\alpha g_c)+(g/g_c)z$. The energy balance is

$$\frac{-\partial[w(\epsilon+pv)]}{\partial x}+Q=A\,\frac{\partial(\rho\epsilon)}{\partial\tau} \quad (2)$$

Introduction of the continuity equation leads to

$$u\frac{\partial}{\partial x}\left(E+\frac{u^2}{2\alpha g_c}+\frac{g}{g_c}z\right)+\frac{\partial}{\partial\tau}\left(E+\frac{u^2}{2\alpha g_c}+\frac{g}{g_c}z\right)+\frac{1}{\rho A}\frac{\partial}{\partial x}(puA)=\frac{Q}{\rho A} \quad (3)$$

which may be written as

$$\frac{D\epsilon}{D\tau}\times\frac{1}{\rho A}\frac{\partial(puA)}{\partial x}=\frac{Q}{\rho A} \quad (4)$$

An alternate form is

$$\frac{D}{D\tau}\left(H+\frac{u^2}{2\alpha g_c}+\frac{g}{g_c}z\right)-\frac{1}{\rho}\frac{\partial p}{\partial\tau}=\frac{Q}{\rho A} \quad (5)$$

The equation for motion for flow in a uniform pipe is

$$\frac{1}{g_c}\left(\frac{\partial u}{\partial\tau}+u\frac{\partial u}{\partial x}\right)+\frac{1}{\rho}\frac{\partial p}{\partial x}+\overline{F}+\frac{g}{g_c}\frac{\partial z}{\partial x}=0 \quad (6)$$

The pipe friction, F, has not yet been given an explicit definition. The general equation of motion must, however, reduce to the simple mechanical energy balance for steady-state conditions. It is therefore assumed that

$$\overline{F}=\frac{f_\tau u^2}{2g_cD} \quad (7)$$

The resultant equation of motion is

$$\frac{1}{g_c}\left(\frac{\partial u}{\partial\tau}+u\frac{\partial u}{\partial x}\right)+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{g}{g_c}\frac{\partial z}{\partial x}+\frac{f_\tau u^2}{2g_cD}=0 \quad (8)$$

It is not necessarily true that f_τ will correspond to the Moody friction factor, as the form of the frictional losses has been only assumed and may itself be in error. Recent experiments (*4*), however, have shown the validity of Equation (8) at least for isothermal flow. These results indicate that f_τ corresponds to the Moody friction factor of steady state when transient velocities from 15 to 75 ft./sec. and accelerations from 0 to 35 ft./sec.^{2} are used. It was reported that the total observed pressure drops could be calculated from steady-state velocity losses and acceleration heads except during periods of initial impulse.

The method of approach in applying the unsteady-state partial-differential equations to loop systems is to reduce the equations of motion and the energy balance to simplified finite-difference equations. Coasting, or isothermal transients, has been considered in reference *1*, and in this section the nonisothermal transient behavior is considered.

The equation of motion, Equation (8), is applied to the schematic loop shown in Figure 4. For this application, F represents the frictional losses which are given by the ordinary steady-state correlations. For the transient case, the thermal loads of the two heat exchangers generally will not be equal and will fluctuate with time. The nonisothermal operation of the convective loop for an all-liquid system would require the addition of a surge tank as shown in dotted lines in Figure 4. It is assumed that the presence of the surge tank will not affect the loop flow except to minimize the pressure fluctuations. The additional assumption made is that the volumetric rate of flow is uniform around the loop at any instant (independent of position). An estimate of the differences in volumetric flow rates in the hot and cold legs is given in reference *1*, and an alternate method based upon the use of an instantaneous constant weight rate of flow also is noted.

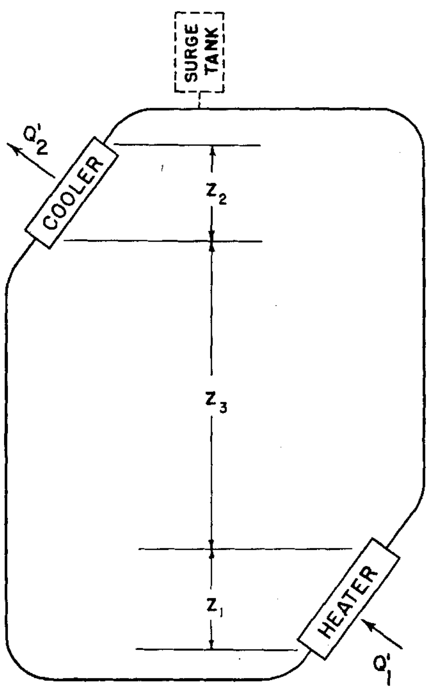

Fig. 4. Natural-circulation loop (schematic).

With the assumption that $\partial V/\partial x=0$ and $\partial V/\partial\tau$ is independent of x, the equation of motion can be simplified to

$$\frac{dV}{d\tau}=\frac{-\int_o \rho dz-\int_o \rho\overline{F}dx}{\int_o \frac{\rho}{g_c}\frac{dx}{A}} \quad (9)$$

Equation (9) may be symbolized as follows:

$$\frac{dV}{d\tau}=\frac{H_l-R_l}{a_l} \quad (10)$$

This equation expresses the volumetric acceleration in terms of the net driving force, which is the difference between the loop hydraulic head and total loop resistance and the loop geometry. The total loop resistance to flow, R_l, is identical with the steady-state values for similar temperatures and rates of flow. H_l corresponds to the loop hydraulic head, and a_l to the loop constant. In order to solve this equation simultaneously with the energy equation, it will be necessary to replace the derivative by its finite-difference approximation. This gives

$$V'=V+\Delta\tau\left[\frac{H'_l-R_l}{a_l}\right] \quad (11)$$

where the prime denotes a forward extrapolation in time. The time interval should be small enough that H_l, R_l, and a_l may be evaluated at either end of the time interval without appreciable effect on the solution. The energy equation, as noted below, permits the evaluation of H_l.

It can be readily ascertained that a suitable form of the energy equation is simply a heat balance.

$$\frac{DH}{D\tau}=C_p\left(\frac{\partial T}{\partial\tau}+u\frac{\partial T}{\partial x}\right)=\frac{Q}{\rho A} \quad (12)$$

Heat conduction along the axis of flow and the mechanical energy terms can be neglected. Replacing the partial derivatives by their approximate finite differences results in

$$\frac{T'_{x+\Delta x} - T_{x+\Delta x}}{\Delta \tau} + \frac{V}{A\,\Delta x}\left(T_{x+\Delta x} - T_x\right) = \frac{Q}{\rho C_p A} \quad (13)$$

or

$$T'_{x+\Delta x} = \left(1 - \frac{V\Delta\tau}{A\,\Delta x}\right) T_{x+\Delta x} + \frac{V\Delta\tau}{A\,\Delta x} T_x + \frac{Q\Delta\tau}{\rho C_p A} \quad (14)$$

This equation determines the change of temperature with time over a volume element of fluid subject to given conditions of rate of flow and of feed input.

The analysis would begin by subdividing the loop into a number, say N, of finite elements. Equation (14) would then be applied to each subdivision with the appropriate value of Q, ρ, A, and Δx to give N equations of the form

$$T'_n = \left(1 - \frac{V\Delta\tau}{\overline{V}_n}\right) T_n + \frac{V\Delta\tau}{\overline{V}_n} T_{n-1} + \frac{Q_n\Delta\tau}{\rho_n C_p A_n} \quad (15)$$

where $\overline{V}_n = A_n\,\Delta x_n$, the volume of the subdivision. For a particular value of the time increment, the right side of this equation is known from initial conditions and method of subdivision. The temperature distribution and therefore the hydraulic head, H'_l, may then be evaluated at the new instant. With the aid of Equation (11) the rate of flow may also be determined at the new instant. Repeated applications of these two equations will produce a record of temperature and rates of flow.

The heat added from the surroundings, Q, will in general be composed of both direct heat addition, such as by an immersion heater, and heat gained through the walls; for example,

$$Q = q + U\pi D\,(t - T)$$

where q = direct heat input (immersion heater, etc.), U = local over-all or film heat transfer coefficient, and t = local external temperature (wall temperature). Equation (15) may be generalized as

$$T_{n,\,m+1} = \left(1 - \beta_n\Delta\tau - \frac{V\Delta\tau}{\overline{V}_n}\right) T_{n,\,m} + \frac{V\Delta\tau}{\overline{V}_n} T_{n-1,\,m} + \beta_n\,\Delta\tau\, t_{n,\,m} + \frac{q_n\,\Delta\tau}{\rho_n C_p A_n} \quad (16)$$

where $\beta_n = \dfrac{U_n\,\pi D}{\rho_n C_p A_n}$ and $\tau = m\Delta\tau$ for m an integer.

The interchange of heat between the fluid and the confining piping needs to be taken into account if the heat capacity of the metal walls is relatively large or if there are significant heat losses from the loop system. For the experimental loop reported in this paper, sufficient insulation was provided so that only the raising and lowering of the pipe-wall temperature needed to be taken into account. A simple heat balance (no longitudinal conduction, no external heat losses, and no radial temperature distributions) for the confining walls of the subdivision yields

$$\frac{dt_n}{d\tau} = \frac{h}{\dfrac{(D_o^2 - D_n^2)}{4D_n}\,\rho_w c_w}\,(t_n - T_n) \quad (17)$$

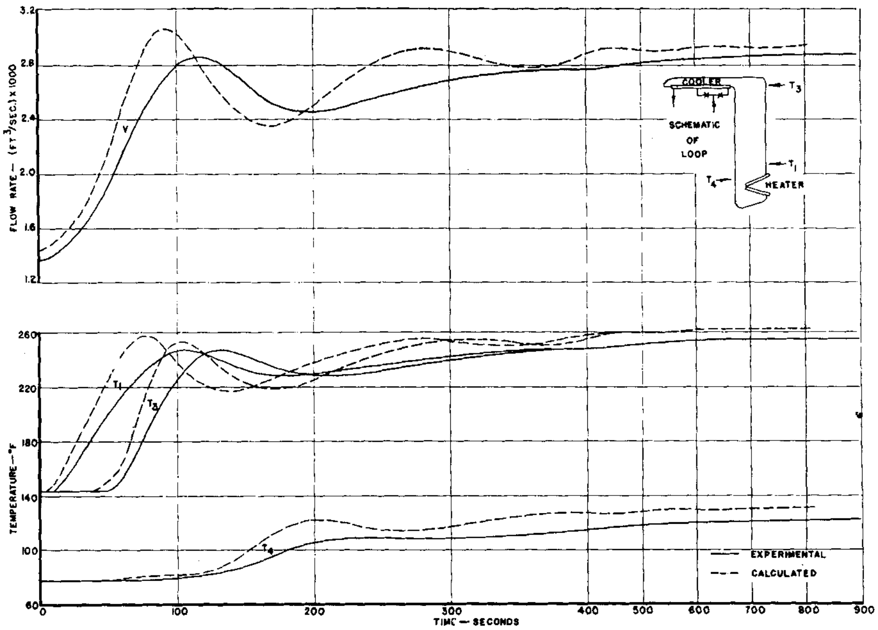

Fig. 5. Comparisons of calculated and experimental transient operations, computer problem 3, first revision.

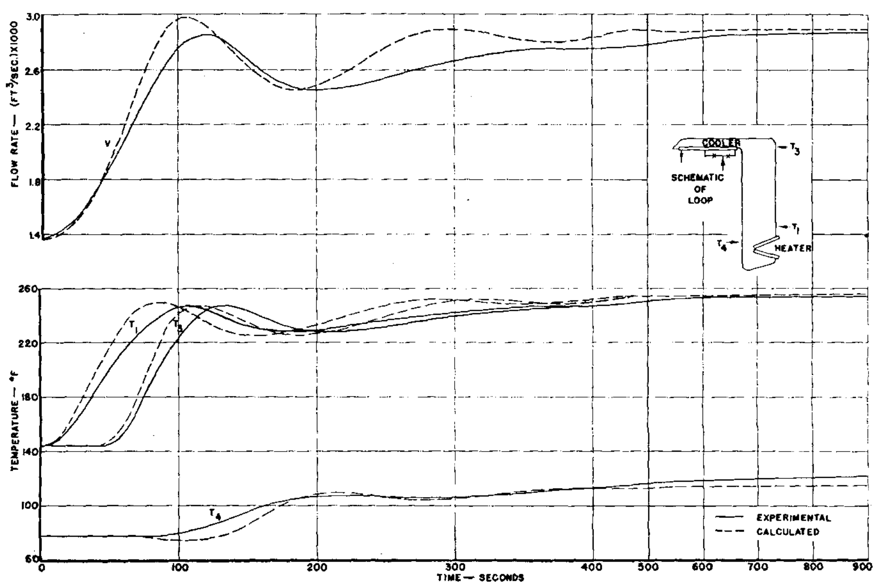

Fig. 6. Comparisons of calculated and experimental transient operations, computer problem 3, second revision.

In finite-difference form Equation (17) is written as

$$t'_n = \left(1 - \frac{h\Delta\tau}{\delta\rho_w c_w}\right) t_n + \frac{h\Delta\tau}{\delta\rho_w c_w} T_n \quad (18)$$

where $\delta = (D_o^2 - D_n^2)/4D_n$ (in feet) approximates the wall thickness.

STABILITY AND CONVERGENCE OF FINITE-DIFFERENCE EQUATIONS

In cases where it is impossible to obtain an exact solution of a partial-differential equation, an approximate solution may be obtained from a corresponding finite-difference equation. The merit of such a numerical solution will then depend upon the convergence of the finite-difference solution to the exact solution. The magnitude of the increments used affects not only the convergence of the numerical solution but also the stability of the solution. Errors may be compounded in the computation and result in instability. The general problem of stability and convergence of numerical solutions has been discussed in detail in the literature(*3, 8, 12, 13, 15, 16, 18, 19*, among others), primarily for the wave and heat-conduction equations. Though the flow equation (11) would be difficult to analyze formally for convergence and stability, a relatively simple means of curbing an error in time and distance for the heat balance leads to the following criteria for stability (*1*):

$$\text{For } b = \frac{V\Delta\tau}{\overline{V}_n} = \text{mesh ratio}$$

$$a = 1 - \beta_n \Delta\tau - b$$

$$|a| \leqslant 1, \ |b| \leqslant 1, \ |a+b| \leqslant 1$$

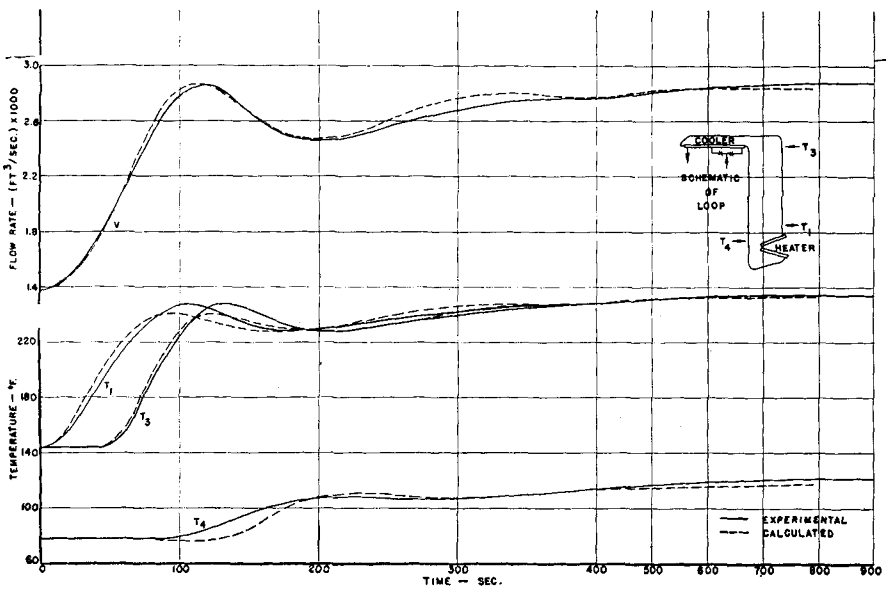

Fig. 7. Comparisons of calculated and experimental transient operations, computer problem 3, third revision.

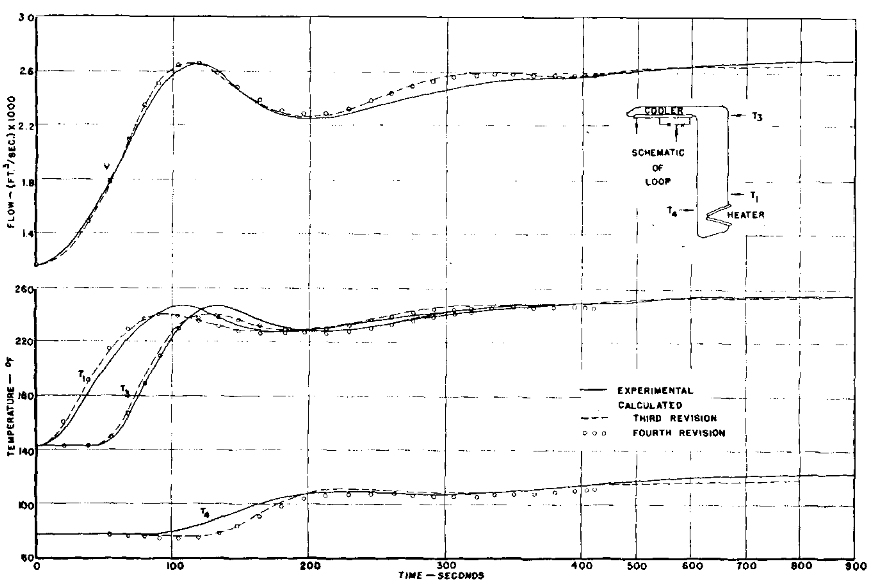

Fig. 8. Comparisons of calculated and experimental transient operations, computer problem 3, fourth revision.

ILLUSTRATION OF FINITE DIFFERENCE EQUATIONS FOR COMPUTER EVALUATIONS

The method of loop subdivision and the evaluations of the constants in the finite-difference equations are illustrated for computer problem 4. This problem concerns the response of the second experimental loop to an abrupt input of heat starting with the fluid at uniform temperature and at rest. The operating conditions are summarized in Table 1. Table 3 summarizes the loop parameters used in the calculations.

Temperatures of the fluid and wall for each subdivision are evaluated through the use of Equations (16) and (18), respectively. A summary of the actual equations employed is given in Table 4. Equation (11) is used to determine the flow. The approximation used for the computation of the hydraulic head, H'_l, is

$$H'_l = \int_o^N \rho' \, dx = \sum_{n=1}^{N} \rho' Z_n$$

where Z_n represents the vertical projection of the subdivision. A negative value is used if the flow is upward in the subdivision, and a positive value for downward flow. The total number of subdivisions is N. The densities are computed from the fluid temperature and the relation

$$\rho_n = 62.404 + 0.4556\left(\frac{T_n}{100}\right) - 0.9605\left(\frac{T_n}{100}\right)^2 + 0.08164\left(\frac{T_n}{100}\right)^8 \quad (19)$$

This density equation was obtained by applying the method of least squares to the tabulated densities of water given in McAdams(*17*) and in the International Critical Tables. The equation is valid for temperatures up to 300°F. and is accurate to 0.01% for low pressures. The evaluation of the loop constant, a_l, is given in Table 3, and the flow equations for viscous and turbulent flow are given in Table 4.

COMPARISON OF EXPERIMENTAL AND CALCULATED TRANSIENT LOOP RUNS

A total of sixty-four transient natural-circulation runs was made on the first loop. These runs served to demonstrate the reproducibility of the tests and to characterize the response to abrupt additions and release of heat to the fluid under various initial conditions. Attention is focused on only two runs on the first loop for purposes of comparing the experimental results with the numerical solution of the finite-difference equations. The conditions of the two runs for computer problems 2 and 3 are listed in Table 1. The progressive refinement of the relations for the loop parameters including a reduction in the number of subdivisions and the effects on the numerical evaluations of computer problem 3 are summarized in Table 2 and illustrated in Figures 5 to 10. The comparisons of calculated and experimental values of flow and temperatures given in Figure 6 indicate that the transient operation of the natural-circulation loop can be predicted with a fair degree of assurance. Several points of departure may be summarized as follows:

1. Difference between the initial maximum values of T_1 and T_3 indicates that the film heat transfer coefficient should be increased.
2. Lag between experimental and calculated temperatures indicates the importance of the heater lag.
3. Difference between initial flow rates indicates more precision required in specifying loop resistance.
4. Difference between steady-state temperatures indicates the need for an improved cooler equation.

The successive improvement of the loop parameters for revisions 2 and 3 and the approach of the calculated values to the experimental values are revealed in Figures 6 and 7. The fourth revision of computer problem 3, given by Figure 8, shows little or no departure from the third revision even though the number of subdivisions in the loop was nearly halved.

Computer problem 2 represents a more exacting test of the analytical equations for the volumetric flow starts at zero. The same finite-difference equations that were successful for the third revision of computer problem 3 were applied in the second revision of computer problem 2. The calculated and experimental results are shown in Figure 9, and, in general, the agreement is satisfying. Further improvement could be achieved by correcting the heat transfer coefficient and refining the viscous-flow loop resistance.

The second loop, with its faster response, provided a relatively severe test of the analytical approach. Computer problem 4 was calculated for operating conditions corresponding to transient run 65, which was made on the second loop. The finite volume subdivisions used were of the same order of magnitude as those employed in the third revision of computer problem 3, and the details of the equations have been previously noted. The comparisons of the calculated and experimental results are shown in Figure 10. The agreement between experimental and predicted values is generally very good. Had the heat transfer coefficient for the cooler been corrected for the num-

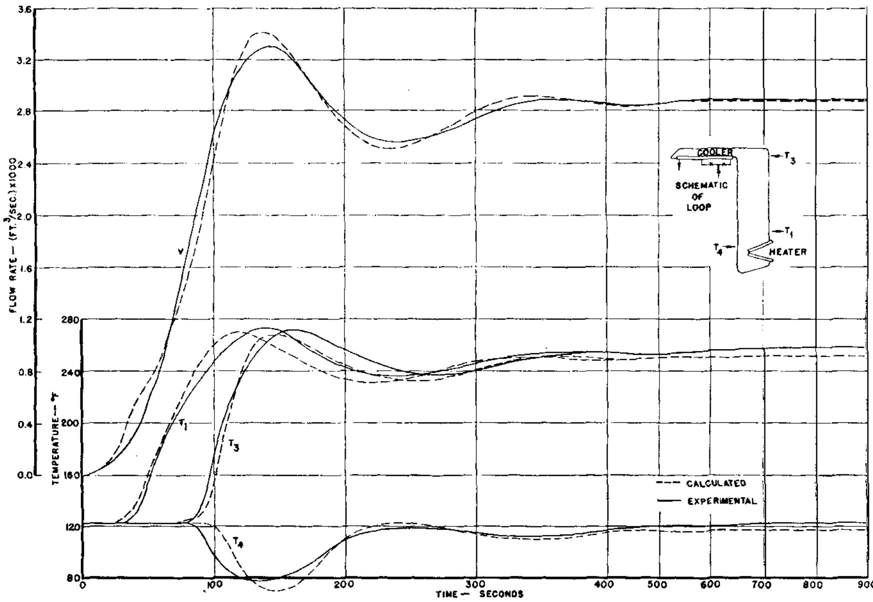

Fig. 9. Comparisons of calculated and experimental transient operations, computer problem 2, first revision.

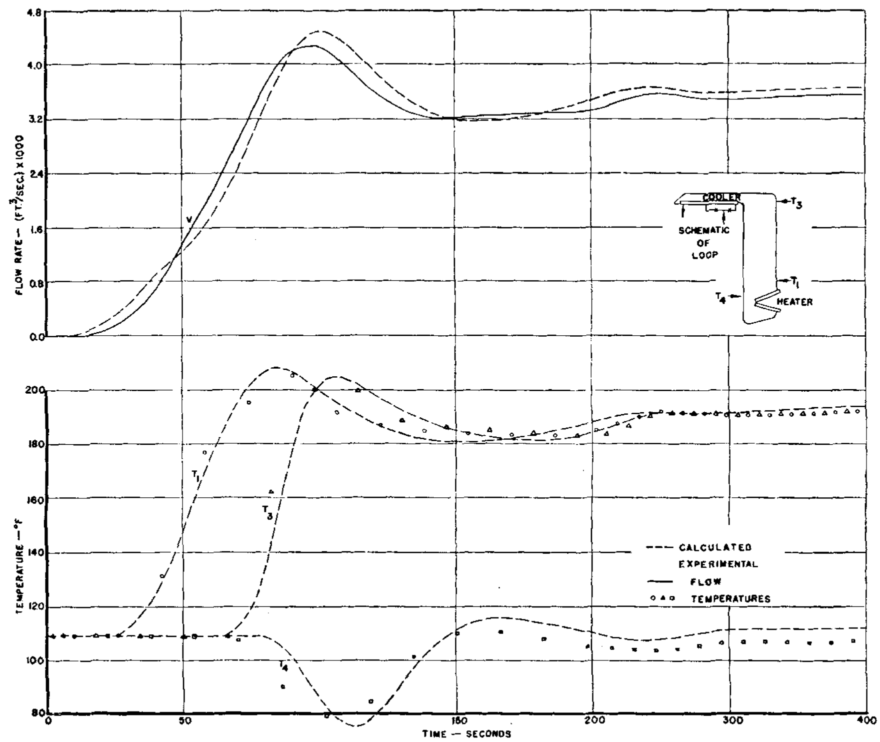

Fig. 10. Comparisons of calculated and experimental transient operations, computer problem 4.

ber of subdivisions, better agreement of steady-state values would have been attained.

SUMMARY

A method for predicting the transient behavior of a single-phase, natural-circulation loop was demonstrated. The method employs the iterative solution of the finite-difference energy and momentum balances and has the following points of interest:

1. The one-dimensional analysis appears to be sufficient for engineering purposes.
2. The assumption of uniform average volumetric flow around the loop at a given instant is sufficiently valid for most expected conditions.
3. The finite-difference equations may be simplified considerably by the use of an average fluid density in the flow equation except for the hydraulic-head term.
4. The degree of reliability of the mathematical model will depend upon the accuracy with which the physical parameters are known or can be estimated; for example, two main factors are loop resistance and heat transferred to the fluid.
5. Relatively large subdivisions may be employed in the finite-difference equations if the proper mesh ratio is chosen.

ACKNOWLEDGMENT

The work described in this paper was made possible by contract AT(11-1)211 between the Atomic Energy Commission and the Chemical Engineering Department of the University of Minnesota. It is a pleasure to acknowledge the encouragement and assistance of J. C. Boyce, Associate Director of the Argonne National Laboratory, in formulating an effective liaison between A.N.L. and the university. Further, the cooperation and assistance of P. C. Ostergaard of the Westinghouse Atomic Power Division is appreciated. The coding of the problem by, and the general assistance of, the National Bureau of Standards are gratefully acknowledged.

NOTATION

a = temperature coefficient = 1-$\beta\Delta\tau$-b, lb.mass/sec.²/ft.⁵
a_l = loop constant = $\int_o \rho/g\, dx/A$
A = cross-sectional area, sq.ft.
b = mesh ratio = $V\Delta\tau/V_n$
$C_p = c$ = specific heat of fluid, B.t.u./(lb.mass) (°F.)
D = diameter, ft.
E = internal energy, B.t.u./lb.mass
ε = total energy = $E + u^2/2\alpha g_c + (g/g_c)z$, B.t.u./lb.mass
f = Moody friction factor
f' = Fanning friction factor
$f\tau$ = friction factor for transient flow (in this paper $f\tau = f$)
F = frictional loss, total, ft.(lb.force)/lb.mass
$\overline{F}$ = frictional loss per unit length, ft.(lb.force)/ft.(lb.mass)
g = local acceleration of gravity, ft./sec.²
g_c = conversion factor in Newton's law of motion, 32.2(lb.mass) ft./(lb.force)sec.²
G = mass rate of flow, lb.mass/(sq.ft.) (sec.)
GPM = coolant rate of flow gal./min.
h = film heat transfer coefficient, B.t.u./(sec.) (sq.ft.) (°F.)
h' = film heat transfer coefficient, B.t.u./(hr.) (sq.ft.) (°F.)
H = enthalpy, B.t.u./lb.mass
H_l = loop hydraulic head = $\int_o \rho dz$, lb.mass/sq.ft.
k = thermal conductivity, B.t.u./(sec.) (sq.ft.) (°F./ft.)
L = length, ft.
m = integer, denoting time position
n = integer, denoting space position
N = number of subdivisions in loop
p = pressure, lb.force/sq.ft.
q = linear rate of heat input to heater element, B.t.u./(ft.) (sec.)
q' = rate of heat input to heater element, B.t.u./sec.
Q_{kw} = rate of heat input to heater element, kw.
Q = linear rate of heat input to fluid, B.t.u./(ft.) (sec.)
Q' = rate of heat input to fluid, B.t.u./sec.
r = pipe radius, ft.
Re = Reynolds' number = $Du\rho/\mu$
R_l = total loop resistance = $\int_o \rho F dx$, lb.mass/sq.ft.
T = fluid temperature, °F.
t = external temperature, wall or coolant, °F.
u = linear velocity, ft./sec.
U = over-all heat transfer coefficient, B.t.u./(sec.) (sq.ft.) (°F.)
v = specific volume = $1/\rho$, cu.ft./lb.mass
V = volumetric rate of flow, cu.ft./sec.
$\overline{V}_n$ = volume of subdivision = $A_n\Delta x_n$, cu.ft.
$\overline{V}_c$ = total cooler volume, cu.ft.
$\overline{V}_e$ = heater element volume, cu.ft.
$V\Delta\tau/V_n$ = mesh ratio
w = mass rate of flow, lb.mass/sec.
x = distance along streamline, ft.
x_n = length of subdivision, ft.
z = vertical projection, ft.
Z_n = vertical projection of subdivision, ft.
α = coefficient in kinetic energy term = ½ for viscous flow = 1 for turbulent flow
$\delta = (D_o^2 - D_n^2)/4D_n \cong$ wall thickness, ft.
μ = viscosity, lb./ft.sec.
μ_{cp} = viscosity, centipoises
ρ = density, lb./cu.ft.
ρc = volumetric heat capacity of fluid = ρc_p, B.t.u./(cu.ft.) (°F.)
$\rho_e c_e$ = volumetric heat capacity of heater element, B.t.u./(cu.ft.) (°F.)
$\rho_w c_w$ = volumetric heat capacity of walls, B.t.u./(cu.ft.) (°F.)
τ = time, sec.
$\int_o$ = integration around closed loop

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(Presented at A.I.Ch.E. New York meeting)